Thematic section

RAGS

Real Algebraic Geometry and Singularities

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SCHEDULE OF THE SECTION
Real Algebraic Geometry and Singularities

- Monday – September 4th

  16:00–16:30 Elías Baro, *Spectral spaces in o-minimal structures*

  16:30–17:00 Carles Bivià-Ausina, *Global multiplicity, special closure and non-degeneracy of gradient maps*

  coffee break

  17:30–18:00 Riccardo Ghiloni, *Subfield-algebraic geometry I: introduction*

  18:00–18:30 José F. Fernando, *Subfield-algebraic geometry II: main results*

  18:30–19:00 Enrico Savi, *Subfield-algebraic geometry III: the* $\mathbb{Q}$*-algebraicity problem*

- Tuesday – September 5th

  14:30–15:00 Antonio Carbone, *Desingularization of semialgebraic sets*

  15:00–15:30 Luis José Santana Sánchez, *Duality of divisors and curves on Mori dream spaces*

  15:30–16:00 Juliusz Banecki, *Extensions of k-regulous functions from two-dimensional varieties*

  coffee break

  16:30–17:00 Wojciech Kucharz, *Approximation and Homotopy in Regulous Geometry*

  17:00–17:30 Stanisław Spodzieja, *Effective Bertini theorem and formulas for multiplicity and the local Łojasiewicz exponent*

  17:30–18:00 Krzysztof Nowak, *Solution to a problem of pulling back singularities*

  18:00–18:30 Tadeusz Krasiński, *Jump of the Milnor number in linear deformations and* δ*-constant families of curve singularities*

- Thursday – September 7th

  14:00–14:30 Marcin Bilski, *Hartogs-type theorems in real algebraic geometry*

  14:30–15:00 Evelia Rosa García Barroso, *Combinatorial study of Morsifications of real univariate singularities*

  15:00–15:30 Janusz Gwoździewicz, *On some regularity condition*

  15:30–16:00 Tomasz Kowalczyk, *On sums of even powers of polynomials*

  coffee break

  16:30–17:00 Krzysztof Kurdyka, *A Bochnak-Siciak theorem for Nash functions over real closed fields*

  17:00–17:30 Maria-Angeles Zurro, *Waring decomposition of real binary forms and Brion's formula*

  17:30–18:00 Wiesław Pawłucki, *Strict* C^p*-triangulation of sets definable in o-minimal structures*

  18:00–18:30 Andrzej Lenarcik, *Remarks on the Łojasiewicz exponent and polar invariants of real curve*

Extensions of k-regulous functions from two-dimensional varieties

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Abstract

Even though Regulous Geometry has already undergone massive development since its invention, it still admits plenty of unsolved fundamental problems. In particular it is unknown whether every k-regulous function on a closed subvariety arises as a restriction of a k-regulous function defined on the entire ambient variety. We discuss cases in which the problem has been settled, in particular we focus on a recent result solving it completely in dimension 2.

[1] Banecki J., *Extensions of k-regulous functions from two-dimensional varieties*, arXiv:2303.02481 (2023).

[2] Fichou G., Monnier J.P., Quarez R., *Continuous functions on the plane regular after one blowing-up*, Mathematische Zeitschrift 285 (2017), no. 1, 287–323.

[3] Kollár J., Nowak K., *Continuous rational functions on real and p-adic varieties*, Mathematische Zeitschrift 279 (2015), no. 1, 85–97.

Spectral spaces in o-minimal structures

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joint work with J. F. Fernando and D. Palacin

Abstract

Normal spectral spaces form a topological context related with several areas of mathematics, specially with Real algebraic geometry. In the late 1970s, the real spectra of a ring was introduced by M. Coste and M.F. Roy as a substitute in real algebraic geometry for the Zariski spectra of a ring in ordinary algebraic geometry. The real spectra associated to a semialgebraic set X is an object whose *standard* topological properties yield information about *semialgebraic* topological properties of X.

Spectral spaces within Model Theory were first considered by A. Pillay, who introduced the so-called *o-minimal spectra* of definable sets, mainly as a tool to develop sheaf theories in the o-minimal setting.

The purpose of this talk is to relate topological properties of the o-minimal spectrum (a normal spectral space) with some relevant model-theoretic notions. Moreover, if time permits we will see that even in abstract model theoretic situations beyond o-minimality it is possible to associate meaningfully a normal spectral space.

Combinatorial study of Morsifications of real univariate singularities

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joint work with A. Bodin, P. Popescu-Pampu and M.S. Sorea

Abstract

We study a broad class of morsifications of germs of univariate real analytic functions. We characterize the combinatorial types of the resulting Morse functions by valuative methods, via planar contact trees constructed from Newton-Puiseux roots of the polar curve of the morsification.

Hartogs-type theorems in real algebraic geometry

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joint work with J. Bochnak and W. Kucharz

Abstract

Let $f : X \to \mathbb{R}$ be a function defined on a connected nonsingular real algebraic set X in $\mathbb{R}^n$. It turns out that regularity of f can be detected by controlling the restrictions of f to algebraic curves or surfaces in X. If $\dim X \geqslant 2$, then f is a regular function whenever $f|_C$ is a regular function for every algebraic curve C in X that is homeomorphic to the unit circle and has at most one singularity. If $\dim X \geqslant 3$, then f is a regular function whenever $f|_S$ is a regular function for every nonsingular algebraic surface S in X that is homeomorphic to the unit 2-sphere.

Global multiplicity, special closure and non-degeneracy of gradient maps

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joint work with J.A.C. Huarcaya
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Abstract

Given a polynomial map $F : \mathbb{C}^n \to \mathbb{C}^p$ with finite zero set, $p \geqslant n$, we introduce the notion of global multiplicity associated to F, which is analogous to the multiplicity of ideals in Noetherian local rings. This notion allows to characterize numerically the Newton non-degeneracy at infinity of F. This fact motivates us to study a combinatorial inequality concerning the normalized volume of global Newton polyhedra and to characterize the corresponding equality using special closures. We also study the Newton non-degeneracy at infinity of gradient maps and discuss some implications of our study with the index of real polynomial vector fields and the estimation of Łojasiewicz exponents at infinity.

Desingularization of semialgebraic sets

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joint work with J.F. Fernando

Abstract

Hironaka's resolution of singularities of algebraic varieties (over a field of characteristic 0) is a widespread celebrated discipline that has many applications in many areas of Mathematics. When the ground field is $\mathbb{R}$ the general approach consists of the following: *Given a real algebraic set $X \subset \mathbb{R}^n$, one finds a non-singular real algebraic set $X' \subset \mathbb{R}^m$ together with a proper polynomial map $f : X' \to X$ that is a biregular isomorphism outside the set of singular points of X.*

If $\mathcal{S} \subset \mathbb{R}^n$ is a semialgebraic set, what does it mean to desingularize $\mathcal{S}$? Unlike the algebraic case, a semialgebraic set $\mathcal{S}$ may have a boundary (that is still a semialgebraic set) and the boundary may have multiple irreducible components. Thus, we must be careful with the choice of the class of 'non-singular semialgebraic domains' to represent semialgebraic sets as the image under proper semialgebraic maps 'as much regular as possible'. If $\mathcal{S}$ is closed we can choose as 'non-singular semialgebraic domains' Nash manifolds with corners and as maps proper polynomial maps. If the involved semialgebraic set $\mathcal{S}$ is not locally closed we have to change the 'non-singular semialgebraic domains' (using Nash quasi-manifolds with corners instead of Nash manifolds with corners) and the involved proper maps are in general only Nash maps instead of polynomial maps.

We also construct the Nash double $D(\mathcal{Q})$ of a Nash manifold with corners $\mathcal{Q} \subset \mathbb{R}^m$, which generalizes the Nash double of a Nash manifold with boundary. This construction and the previous result allow, when $\mathcal{S}$ is closed, to obtain a branched Nash covering $f : D(\mathcal{Q}) \to \mathcal{S}$ whose fibers have constant cardinality outside of the ramification locus.

If time permits, we will show several applications of our results:

- Nash approximation of continuous semialgebraic maps whose target spaces are Nash manifolds with corners.
- Representation of compact semialgebraic sets connected by analytic paths as images under Nash maps of closed unit balls.

- Weak desingularization of closed semialgebraic sets using Nash manifolds with (smooth) boundary.
- Explicit construction of Nash models for compact orientable smooth surfaces.

Subfield-algebraic geometry II: main results

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Abstract

Let L be either an algebraically closed field of characteristic zero or a real closed field, and let K be any subfield of L. Let $\overline{K}^{\blacksquare}$ be the algebraic closure of K in the first case and the real closure of K in the second case. We identify $\overline{K}^{\blacksquare}$ with the algebraic closure of K in L. Given an algebraic set $X \subset L^n$ that can be described using finitely many polynomials of $\overline{K}^{\blacksquare}[\mathtt{x}_1, \ldots, \mathtt{x}_n]$ we explain a procedure to compute geometrically the smallest K-algebraic subset of L^n that contains X, that is, the smallest algebraic subset of L^n that contains X and can be described using finitely polynomials of $K[\mathtt{x}_1, \ldots, \mathtt{x}_n]$. In 1974 Stengle approached a similar problem (from the algebraic point of view) when L is a real closed field and K is endowed with the unique ordering induced by L in K. He provided a real Nullstellensatz to compute the zero ideal (in $L[\mathtt{x}_1, \ldots, \mathtt{x}_n]$) for algebraic sets $X \subset L^n$ that can be described using polynomials with coefficients in K. We will pay special attention to those algebraic subsets of L^n whose zero ideal in $L[\mathtt{x}_1, \ldots, \mathtt{x}_n]$ can be generated by finitely many polynomials with coefficients in K and we will show that this class is very restrictive (due to the strong properties they have). If time allows it we will state some properties concerning $L|K$-regularity and $L|K$-Jacobian criterion of K-algebraic sets.

Subfield-algebraic geometry I: introduction

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Abstract

Let L be either an algebraically closed field of characteristic zero or a real closed field, and let K be any subfield of L. Given a subset X of L^n, we say that $X \subset L^n$ is a K-algebraic set if it is the zero set in L^n of a family of polynomials in the subring $K[\mathtt{x}] = K[\mathtt{x}_1, \ldots, \mathtt{x}_n]$ of $L[\mathtt{x}]$. We are interested in studying the algebraic geometry of K-algebraic sets $X \subset L^n$ using only polynomials in $K[\mathtt{x}]$, and comparing it with the usual algebraic geometry of $X \subset L^n$ in which polynomials of the entire ring $L[\mathtt{x}]$ are used. This study generates a new algebraic geometry, we call subfield-algebraic geometry, which is particularly rich and interesting in the case L is a real closed field and K is not a real closed subfield of L, such as $K = \mathbb{Q}$. We present here some basic notions, results and examples, such as the K-Zariski topology, the K-irreducible components and the K-dimension, and compare them with the usual ones.

On some regularity condition

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joint work with B. Gryszka

Abstract

Let $\mathbb{K}$ be an uncountable field of characteristic zero and let f be a function from $\mathbb{K}^n$ to $\mathbb{K}$. We show that if the restriction of f to every affine plane $L \subset \mathbb{K}^n$ is regular, then f is a regular function.

On sums of even powers of polynomials

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joint work with Julian Vill

Abstract

Let n be a positive integer. For a (commutative) ring A we define its n-th Pythagoras number $p_n(A)$ as the smallest positive integer g such that any sum of n-th powers in A can be expressed as a sum of at most g n-th powers in A. If such number does not exist we put $p_n(A) = \infty$.

During the talk I will discuss main differences between quadratic forms and forms of higher degree. Then, I will show that

$$p_{2d}(\mathbb{R}[x_1, \ldots, x_n]) = +\infty$$

provided that $n \geqslant 2$ and $d \geqslant 1$. This partially solves Problem 8 from [1]. I will also discuss the case of a single variable as well as the ring of formal power series in 2 variables.

[1] Choi M-D., Dai Z.D., Lam T.Y., Reznick B., *The Pythagoras number of some affine algebras and local algebras*, Journal für die reine und angewandte Mathematik 336 (1982), 45–82.

Jump of the Milnor number in linear deformations and δ-constant families of curve singularities

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joint result with A. Zakrzewska

Abstract

The jump of the Milnor number of an isolated singularity f_0 is the minimal non-zero difference between the Milnor numbers of f_0 and a generic element of its deformations. We characterize plane curve singularities for which this jump is equal to one in the class of linear deformations. We use this result to determine topological types of singularities in δ-constant linear deformations in some classes of singularities.

Approximation and Homotopy in Regulous Geometry

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Abstract

Let X, Y be nonsingular real algebraic sets. A map $\varphi\colon X \to Y$ is said to be k-regulous, where k is a nonnegative integer, if it is of class $\mathcal{C}^k$ and the restriction of φ to some Zariski open dense subset of X is a regular map. Assuming that Y is uniformly rational, and $k \geqslant 1$, we prove that a $\mathcal{C}^\infty$ map $f\colon X \to Y$ can be approximated by k-regulous maps in the $\mathcal{C}^k$ topology if and only if f is homotopic to a k-regulous map. The class of uniformly rational real algebraic varieties includes spheres, Grassmannians and rational nonsingular surfaces, and is stable under blowing up nonsingular centers. Furthermore, taking $Y = \mathbb{S}^p$ (the unit p-dimensional sphere), we obtain several new results on approximation of $\mathcal{C}^\infty$ maps from X into $\mathbb{S}^p$ by k-regulous maps in the $\mathcal{C}^k$ topology, for $k \geqslant 0$.

A Bochnak-Siciak theorem for Nash functions over real closed fields

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joint work with W. Kucharz

Abstract

Let R be a real closed field. We prove that if R is uncountable, then a function $f\colon U \to R$ defined on an open semialgebraic set U in R^n, with $n \geqslant 2$, is a Nash function whenever for every affine 2-plane Q in R^n the restriction $f|_{U\cap Q}$ is a Nash function (some condition on the shape of U is required if R is not Archimedean). This is an analog of the Bochnak–Siciak theorem established in the real analytic setting. We also provide an example showing that uncountability of R is essential.

Remarks on the Łojasiewicz exponent and polar invariants of real curve

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Abstract

Let $f \in \mathbb{R}\{X, Y\}$ be a convergent series without constant term such that the analytic function $f : (\mathbb{R}^2, 0) \to (\mathbb{R}, 0)$ defined by him has isolated zero. Let us consider a generic polar curve $\partial f = a(\partial f/\partial X) + b(\partial f/\partial Y) = 0$ ($a, b \in \mathbb{R}$). To every real branch of ∂f with parametrization $\gamma : (\mathbb{R}, 0) \to (\mathbb{R}^2, 0)$ we assign the *real polar invariant*

$$\frac{\operatorname{ord}(f \circ \gamma)}{\operatorname{ord} \gamma} .$$

By $Q^{\mathbb{R}}(f)$ we denote the set of all real polar invariants of f. Gwoździewicz [1], [2] showed that $Q^{\mathbb{R}}(f) \neq \varnothing$. We define the *Łojasiewicz exponent* of f as

$$\begin{aligned} l_0(f) \quad = \quad & \inf\{\lambda > 0 : |f(x,y)| \geqslant C \max\{|x|, |y|\}^{\lambda} \\ & \text{near } \ 0 \in \mathbb{R}^2 \quad \text{for } \ C > 0\}. \end{aligned}$$

Gwoździewicz proved that $l_0(f) = \max Q^{\mathbb{R}}(f)$. We can treat the series $f(X, Y)$ as an element of the ring $\mathbb{C}\{X, Y\}$. Then we can consider the analogous set $Q^{\mathbb{C}}(f)$ of *complex polar invariants* [3]. Obviously $Q^{\mathbb{R}}(f) \subset Q^{\mathbb{C}}(f)$. Moreover, $\max Q^{\mathbb{C}}(f) < \infty$ if and only if f has no multiple factors in $\mathbb{C}\{X, Y\}$.

Results describing relations between $Q^{\mathbb{R}}(f)$ and $Q^{\mathbb{C}}(f)$ will be presented. The techniques as Newton diagrams and characteristics of branches will be applied.

[1] Gwoździewicz J., *Wykładnik Łojasiewicza funkcji analitycznej o zerze izolowanym*, Uniwersytet Jagielloński, PhD thesis, 1995.

[2] Gwoździewicz J., *The Łojasiewicz eksponent of an analytic function at an isolated zero*, Commentarii Mathematici Helvetici 74 (1999), 364–375.

[3] Teissier B., *Variétés polaires*, Inventiones Mathematicae 40 (1977), 267–292.

Solution to a problem of pulling back singularities

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Abstract

The aim of my talk is to give an affirmative general solution to the following pullback problem. Consider a finite holomorphic map germ $\varphi : (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ and an analytic subvariety germ X in the target. Then if the preimage $Y = \varphi^{-1}(X)$, taken with the reduced structure, is smooth, so is X. The case, where Y is not contained in the ramification divisor Z of φ, was established by Ebenfelt–Rothschild (2007) and afterwards by Lebl (2008) and Denkowski (2016). The hypersurface case was achieved by Giraldo–Roeder (2020) and recently by Jelonek (2023).

Strict C^p-triangulation of sets definable in o-minimal structures

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Abstract

Let R be any real closed field expanded by some o-minimal structure. Let $f : A \to R^d$ be a definable and continuous mapping defined on a definable, closed, bounded subset A of R^n. Let p be any positive integer. The main result tells that then there exists a finite simplicial complex $\mathcal{T}$ in R^n and a definable homeomorphism $h : |\mathcal{T}| \to A$, where $|\mathcal{T}| := \bigcup \mathcal{T}$, such that for each simplex $\Delta \in \mathcal{T}$, the restriction of h to its relative interior $\mathring{\Delta}$ is a C^p-embedding of $\mathring{\Delta}$ into R^n and moreover both h and $f \circ h$ are of class C^p in the sense that they have definable C^p-extensions defined on an open definable neighborhood of $|\mathcal{T}|$ in R^n. We call a pair $(\mathcal{T}, h)$ *a strict C^p-triangulation of A*.

Duality of divisors and curves on Mori dream spaces

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Abstract

Let X be a normal $\mathbb{Q}$-factorial variety with zero irregularity. For any integer $0 \leqslant k \leqslant n-1$, we consider the cone D_k of numerical classes of effective divisors on X whose stable base locus has codimension larger than k and we set $\mathcal{D}_k$ to be its closure in $\mathrm{N}^1(X)_{\mathbb{R}}$. We obtain the filtration

$$\mathrm{N}ef(X) = \mathcal{D}_{n-1} \subseteq \cdots \subseteq \mathcal{D}_1 \subseteq \mathcal{D}_0 = \overline{\mathrm{Eff}(X)}.$$

We consider an analogous filtration of cones of pseudo-effective curves:

$$\overline{\mathrm{N}E(X)} = \mathcal{C}_{n-1} \supseteq \cdots \supseteq \mathcal{C}_1 \supseteq \mathcal{C}_0,$$

where $\mathcal{C}_k$ is the closure of the cone in $N_1(X)_{\mathbb{R}}$ generated by classes of *k-moving* curves, that is, classes of irreducible curves moving in a family that sweeps out an $(n-k)$-dimensional subvariety of X. It is known that $\mathcal{C}_{n-1}$ and $\mathcal{C}_0$ are respectively dual to $\mathcal{D}_{n-1}$ and $\mathcal{D}_0$ under the standard intersection product. In general, duality for intermediate cones is not expected. However, [2] proved that for toric varieties, we obtain duality if we also consider curves sweeping out varieties in some small modification of X. We call this *weak duality.* Later, [1] proved that this weak duality holds for Mori dream spaces and asked if, as the pseudo-effective cone, the $\mathcal{D}_k$ cones are also polyhedral.

In this talk we give a positive answer to Choi's question, by relating the filtration of $\overline{\mathrm{Eff}(X)}$ to the Mori chamber decomposition of X. We also present examples where a *strong duality* holds.

[1] Choi S.R., *Duality of the cones of divisors and curves*, Mathematical Research Letters 19 (2012), no. 2, 403–416.

[2] Payne S., *Stable base loci, movable curves, and small modifications, for toric varieties*, Mathematische Zeitschrift 253 (2006), no. 2, 421–431.

Subfield-algebraic geometry III: the $\mathbb{Q}$-algebraicity problem

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Abstract

In 2020, Parusiński and Rond proved that every algebraic set $V \subset \mathbb{R}^n$ is homeomorphic to a $\overline{\mathbb{Q}}^r$-algebraic set $V' \subset \mathbb{R}^n$, where $\overline{\mathbb{Q}}^r$ denotes the field of real algebraic numbers. The aim of this talk is to provide some classes of algebraic sets that positively answer the following open problem:

$\mathbb{Q}$-ALGEBRAICITY PROBLEM: (Parusiński, 2022) Is every algebraic set $V \subset \mathbb{R}^n$ homeomorphic to some $\mathbb{Q}$-algebraic set $V' \subset \mathbb{R}^m$, with $m \geqslant n$?

We introduce the notion of $\mathbb{Q}$-determined $\mathbb{Q}$-algebraic sets. Roughly speaking, a $\mathbb{Q}$-algebraic set is $\mathbb{Q}$-determined if its local behaviour at nonsingular points is described by polynomial equations with rational coefficients via the $\mathbb{R}|\mathbb{Q}$-Jacobian criterion of $\mathbb{Q}$-algebraic sets. This notion is crucial to develop $\mathbb{Q}$-algebraic approximation techniques and to provide a version over $\mathbb{Q}$ of the relative Nash-Tognoli theorem. Latter result, combined with a version over $\mathbb{Q}$ of the classical blowing down lemma, allows us to give a complete positive answer to the above $\mathbb{Q}$-ALGEBRAICITY PROBLEM in the case of nonsingular algebraic sets and algebraic sets with isolated singularities.

Effective Bertini theorem and formulas for multiplicity and the local Łojasiewicz exponent

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Abstract

The classical Bertini theorem on generic intersection of an algebraic set with hyperplanes states the following:

Let X be a nonsingular closed subvariety of $\mathbb{P}_k^n$, where k is an algebraically closed field. Then there exists a hyperplane $H \subset \mathbb{P}_k^n$ not containing X and such that the scheme $H \cap X$ is regular at every point. Furthermore, the set of hyperplanes with this property forms an open dense subset of the complete linear system $|H|$ considered as a projective space.

We show that one can effectively indicate a finite family of hyperplanes H such that at least one of them satisfies the assertion of the Bertini theorem, provided the characteristic of the field k is equal to zero. As an application of the method used in the proof we give effective formulas for the multiplicity and the Łojasiewicz exponent of polynomial mappings.

Waring decomposition of real binary forms and Brion's formula

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joint work with M. Ansola and A. Diaz-Cano

Abstract

The Waring Problem over polynomial rings studies the decomposition of a homogeneous polynomial $p \in K[x_1, \ldots, x_n]$ of degree d in the variables $x_1, \ldots, x_n$ as a linear combination of d-th powers of linear forms with coefficients in the field K. The case of real or complex coefficients is especially important in applications.

In the talk we will focus on the case $n = 2$ where there is a very efficient algorithm to calculate such decompositions, see [1]. We will show how the use of such decompositions allows to obtain a parametric integration formula over triangles in semi-algebraic neighborhoods of p for a certain topology in the space of real binary forms of degree d. This formula is the extension of the formulas given in [2], where the parametric behavior of the integrals was not studied.

[1] Ansola M., DĂaz-Cano A., Zurro M.A., *Semialgebraic sets and real binary forms decompositions*, Journal of Symbolic Computation 107 (2021), 209–220.

[2] Baldoni V., Berline N., De Loera J.A., Köppe M., Vergne M., *How to integrate a polynomial over a simplex*, Mathematics of Computation 80 (2011), 297–325.