Seminario de Álgebra, Geometría algebraica y Singularidades - ULL
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On sensitivity in Cayley graphs

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Recently, Huang proved the Sensitivity Conjecture, by showing that every set of more than half the vertices of the d-dimensional hypercube Q_d induces a subgraph of maximum degree at least $\sqrt{d}$. This is tight by a result of Chung, Füredi, Graham, and Seymour. Huang asked whether similar results can be obtained for other highly symmetric graphs.

We show that high symmetry alone does not guarantee similar behavior. We present three infinite families of Cayley graphs of unbounded degree that contain induced subgraphs of maximum degree 1 on more than half the vertices. In particular, this refutes a conjecture of Potechin and Tsang, for which first counterexamples were shown recently by Lehner and Verret. The first family consists of dihedrants. The second family are star graphs, these are edge-transitive Cayley graphs of the symmetric group. All members of the third family are d-regular containing an induced matching on a $\frac{d}{2d-1}$-fraction of the vertices. This is largest possible and answers a question of Lehner and Verret.

On the positive side, we consider Cayley graphs of Coxeter groups, where a lower bound similar to Huang's can be shown. A generalization of the construction of Chung, Füredi, Graham, and Seymour shows that this bound is tight for products of Coxeter groups of type $\mathbf{A_n}$, $\mathbf{I_2}(2k+1)$, and most exceptional cases and not far from optimal in general. Then, we show that also induced subgraphs on more than half the vertices of Levi graphs of projective planes and of the Ramanujan graphs of Lubotzky, Phillips, and Sarnak have unbounded degree. This yields more classes of Cayley graphs with properties similar to the ones provided by Huang's results. However, in contrast to Coxeter groups these graphs have no large subcubes.

Joint work with Ignacio García-Marco.

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