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Geometría algebraica: singularidades e invariantes

Seminario de Álgebra, Geometría algebraica y Singularidades
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On the relative size of toric bases

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Let $A = \{\mathbf{a}_1, \ldots, \mathbf{a}_m\} \subseteq \mathbb{N}^n$ be a nonzero vector configuration in $\mathbb{Q}^n$ and $\mathbb{N}A := \{l_1\mathbf{a}_1 + \cdots + l_m\mathbf{a}_m \mid l_i \in \mathbb{N}\}$ the corresponding affine semigroup. We grade the polynomial ring $\mathbb{K}[x_1, \ldots, x_m]$ over an arbitrary field $\mathbb{K}$ by the semigroup $\mathbb{N}A$ setting $\deg_A(x_i) = \mathbf{a}_i$ for $i = 1, \ldots, m$. For $\mathbf{u} = (u_1, \ldots, u_m) \in \mathbb{N}^m$, we define the *$A$-degree* of the monomial $\mathbf{x}^\mathbf{u} := x_1^{u_1} \cdots x_m^{u_m}$ to be

$$\deg_A(\mathbf{x}^\mathbf{u}) := u_1\mathbf{a}_1 + \cdots + u_m\mathbf{a}_m \in \mathbb{N}A,$$

while we denote the usual degree $u_1 + \cdots + u_m$ of $\mathbf{x}^\mathbf{u}$ by $\deg(\mathbf{x}^\mathbf{u})$. The *toric ideal I_A* associated to A is the prime ideal generated by all the binomials $\mathbf{x}^\mathbf{u} - \mathbf{x}^\mathbf{v}$ such that $\deg_A(\mathbf{x}^\mathbf{u}) = \deg_A(\mathbf{x}^\mathbf{v})$.

We consider the Graver basis, the universal Gröbner basis, a Markov basis and the set of the circuits of a toric ideal. We present several theorems concerning bounds on the size of these bases or the maximal degree of their elements in terms of the size or the maximal degree of the other bases.

Joint work with A.Thoma.

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