



Seminario de Álgebra, Geometría algebraica y Singularidades
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Gluing semigroups in \mathbb{N}^n : when and, sometimes, how

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A semigroup $\langle C \rangle$ in \mathbb{N}^n is a gluing of $\langle A \rangle$ and $\langle B \rangle$ if its finite set of generators C splits into two parts, $C = k_1A \sqcup k_2B$ with $k_1, k_2 \geq 1$, and the defining ideals of the corresponding semigroup rings satisfy that I_C is generated by $I_A + I_B$ and one extra element. Two semigroups $\langle A \rangle$ and $\langle B \rangle$ can be glued if there exist positive integers k_1, k_2 such that, for $C = k_1A \sqcup k_2B$, $\langle C \rangle$ is a gluing of $\langle A \rangle$ and $\langle B \rangle$.

Although any two numerical semigroups, namely semigroups in dimension $n = 1$, can always be glued, it is no longer the case in higher dimensions. In this talk, we will give necessary and sufficient conditions on A and B for the existence of a gluing of $\langle A \rangle$ and $\langle B \rangle$, and give examples to illustrate why they are necessary. These generalize and explain the previous known results on existence of gluing. We also show that the glued semigroup $\langle C \rangle$ inherits the properties like Gorenstein or Cohen-Macaulay from the two parts $\langle A \rangle$ and $\langle B \rangle$.

This talk is based on a series of joint papers with Hema Srinivasan (University of Missouri, Columbia, EE.UU.).

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