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Geometría algebraica: singularidades e invariantes

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# The Łojasiewicz exponent of the gradient of plane complex curve with respect to its polar curve

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Let $f = 0$ be a germ of an isolated singularity at $0 \in \mathbb{C}^2$ defined by $f \in \mathbb{C}\{X, Y\}$. The Łojasiewicz exponent $\mathcal{L}_0(f)$ is the best $\theta \geq 0$ in the local inequality

$$(*) \quad |\mathrm{grad} f(z)| \geq c|z|^{\theta}.$$

Every nonsingular $\lambda \in \mathbb{C}\{X, Y\}$ (local parameter) defines the polar curve $\Gamma_{f,\lambda}$: $(\partial\lambda/\partial X)(\partial f/\partial Y) - (\partial\lambda/\partial Y)(\partial f/\partial X) = 0$. Restricting (*) to $z \in \Gamma_{f,\lambda}$ we consider the relative Łojasiewicz exponent $\mathcal{L}_0(f|\Gamma_{f,\lambda})$. After [1, 2, 3] we know that $\mathcal{L}_0(f) = \mathcal{L}_0(f|\Gamma_{f,\lambda})$ when $f$ and $\lambda$ are transverse. Therefore we focus mostly on the case with $f$ and $\lambda$ tangent. The relative Łojasiewicz exponent does not change after analytical coordinate change. We ask whether or not this exponent is an equisingularity invariant of the pair $f, \lambda$. We found specific equisingularity classes with negative answer. These classes may be described by using Newton polygon. After coordinate change with $\lambda = X$ (in this case $\Gamma_{f,\lambda} = \{\frac{\partial f}{\partial Y} = 0\}$), the Newton polygon of $f$ has one segment that joins $(p, 0)$ and $(0, q)$ with $\mathrm{ord1}\, f = p < q$. Moreover $f$ is nondegenerate on this segment. In such class we state that $\mathcal{L}_0(f|\frac{\partial f}{\partial Y} = 0)$ is greater than or equal to $p - 1$ and less than or equal to $q - 1$. We found examples with different exponents in the same class. For other classes the relative Łojasiewicz exponent is a equisingularity invariant. We describe its value by using a version of the Eggers tree of $f$ and by observing where $\lambda$ leaves the tree.

## Referencias

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