Seminario de Álgebra, Geometría algebraica y Singularidades
La Laguna, 14 de mayo de 2024, 16:00 horas (GMT+1)

Algebraic Geometry codes in the sum-rank metric

Elena Berardini
Université de Bordeaux[1]

Linear codes in the Hamming metric have been playing a central role in the theory of error correction since the 50s and were extensively studied. In reverse, the theory of codes in the sum-rank metric is still in its beginnings, and to date only a few constructions are known.

Algebraic Geometry codes in the Hamming metric allow to overcome the main drawback of Reed-Solomon codes, which is that their length is bounded by the cardinality of the finite field we work on, while benefiting from good parameters. The counterpart of Reed-Solomon codes in the sum-rank metric are linearized Reed-Solomon codes [2]. They have optimal parameters but suffer from the same limitation as Reed-Solomon codes. However, in contrast with the situation of codes in the Hamming metric, no geometric construction has been proposed so far.

In this talk, we will present the first geometric construction of sum-rank metric codes, which we called linearized Algebraic Geometry codes [1]. After introducing some background on codes in the sum-rank metric, we will develop the theory of Riemann-Roch spaces over Ore polynomial rings with coefficients in the function field of a curve, by exploiting the classical theory of divisors and Riemann-Roch spaces on algebraic curves. With this theory at hand, we will study the parameters of linearized Algebraic Geometry codes and give lower bounds for their dimension and minimum distance. Notably, we will show that our new codes exhibit quite good parameters, respecting a similar bound to Goppa's bound for Algebraic Geometry codes in the Hamming metric. Furthermore, our construction yields codes asymptotically better than the sum-rank version of the Gilbert-Varshamov bound.

The talk is based on a joint work with X. Caruso.

References

[1] E. Berardini and X. Caruso. Algebraic Geometry codes in the sum-rank metric. To appear in IEEE Transactions on Information Theory (2024).

[2] U. Martínez-Peñas. Skew and linearized Reed-Solomon codes and maximum sum rank distance codes over any division ring. In: Journal of Algebra 504 (2018), pp. 587-612.

[1]CNRS, IMB
Université de Bordeaux
Bordeaux, Francia
elena.berardini@math.u-bordeaux.fr