Seminario de Álgebra, Geometría algebraica y Singularidades
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The Markov complexity of book graphs

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For $A \in \mathbb{Z}^{m\times n}$ with $\ker_{\mathbb{Z}}(A) \cap \mathbb{N}^n = \{\mathbf{0}\}$ and $r \geq 2$, the r–th *Lawrence lifting* $A^{(r)}$ of A is the $(rm+n) \times rn$ matrix

$$A^{(r)} = \begin{pmatrix} \overbrace{\begin{matrix} A & 0 & & 0 \\ 0 & A & & 0 \\ & & \ddots & \\ 0 & 0 & & A \end{matrix}}^{r-\text{times}} \\ \begin{matrix} I_n & I_n & \cdots & I_n \end{matrix} \end{pmatrix}.$$

We identify an element of $\ker_{\mathbb{Z}}(A^{(r)})$ with an $r \times n$ matrix: each row of this matrix corresponds to an element of $\ker_{\mathbb{Z}}(A)$, and the sum of its rows is zero. The *type* of an element of $\ker_{\mathbb{Z}}(A^{(r)})$ is the number of nonzero rows of this matrix. The *Markov complexity*, $m(A)$, is the largest type of any element in the universal Markov basis of $A^{(r)}$ as r varies.

In this work we compute the exact value of the Markov complexity for toric ideals of book graphs with k expandable pages. In graph theory a tree is said to be starlike if it has exactly one vertex of degree greater than two. This vertex is called the root and a starlike tree is obtained by attaching $k \geq 3$ paths to the root vertex. A book graph with k expandable pages is the cartesian product of a starlike tree and a single edge, and we will call it for short *k-book graph*. Book graphs are ring graphs, thus their toric ideals are complete intersections. We provide the only known family of toric ideals of arbitrarily large Markov complexity that we know explicitly the complexity of each member and a minimal system of generators for each of their higher Lawrence liftings.

This is a joint work with A.Thoma and M.Vladoiu.

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