



Seminario de Álgebra, Geometría algebraica y Singularidades
La Laguna, 6 de mayo de 2025, 15:00 horas (GMT+1)

The invariant ring of pairs of matrices

Xabier García Martínez
Universidade de Vigo¹

Let us consider the action of the general linear group $\mathrm{GL}_n(\mathbb{C})$ on the direct product $\mathcal{M}_n^d$ of d copies of $\mathcal{M}_n$ by simultaneous conjugation sending $(X_1, \dots, X_d)$ to $(gX_1g^{-1}, \dots, gX_dg^{-1})$ for any $g \in \mathrm{GL}_n(\mathbb{C})$. This induces an action of $\mathrm{GL}_n(\mathbb{C})$ on the algebra $\mathbb{C}[\mathcal{M}_n^d]$ of polynomial functions on $\mathcal{M}_n^d$. The algebra of invariants under this action, $\mathbb{C}[\mathcal{M}_n^d]^{\mathrm{GL}_n}$, is an important object in several areas of mathematics.

In this talk we will explain how we used methods coming from non-associative algebras and from computational algebra to obtain the full description of the case $\mathbb{C}[\mathcal{M}_4^2]^{\mathrm{GL}_4}$, which could not be solved using the standard representation theory methods. Moreover, we will talk about its connection with the Calogero-Moser spaces and the Hilbert scheme of points.

Referencias

- [1] F. Eshmatov, X. García-Martínez and R. Turdibaev. Noncommutative Poisson structure and invariants of matrices *Advances in Mathematics*, 469, 110212, (2025).
- [2] F. Eshmatov, X. García-Martínez, T. Normatov and R. Turdibaev. On the coordinate rings of Calogero-Moser spaces and the invariant commuting variety of a pair of matrices. *Results in Mathematics*, 80, Article 68, (2025).
- [3] X. García-Martínez, T. Normatov and R. Turdibaev. The ring of invariants of pairs of 3×3 matrices. *Journal of Algebra*, 603(1), 201–212, (2022).

¹Universidade de Vigo
Vigo, España
xabier.garcia.martinez@uvigo.gal